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The Leontief Paradox Redux

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Abstract

Shortly after Leamer (1980) found that the Leontief Paradox was based on a simple conceptual misunderstanding, Brecher and Choudhri (1982) argued that the fact that the United States exported labor services was, in itself, paradoxical because it is true if and only if its per-capita consumption is less than the world average. Surprisingly, however, no formal answer to this paradox has been provided for nearly four decades. This paper revisits this paradox and formally shows that the paradox can be resolved if the Heckscher–Ohlin– Vanek model takes into account technology differences across countries and trade imbalance. In contrast, the paradox cannot be resolved even if the analysis takes into account quasi-homothetic preferences, the Armington home bias, or offshoring.

Key words: Leontief Paradox, Technology differences, Trade imbalance, Nonhomothetic preferences, Armington home bias, Offshoring

JEL classification code: F14

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"In sum, Leontief's results about the direction of US trade in labor services cannot be reconciled with Vanek's version of the Heckscher-Ohlin model. ... the Leontief Paradox is still with us." — Brecher and Choudhri (1982, p.823)

1 Introduction

Leontief (1953) found that US imports embodied a higher ratio of capital to labor than US exports, using data for 1947. Under the presumption that the US was capital-abundant in 1947, this appears to contradict the Heckscher–Ohlin (HO) theorem which predicts that US exports would be relatively capital intensive. This finding came to be called the "Leontief Paradox", one of the most famous paradoxes in economics. Subsequent empirical research reconsidered the analysis, while taking into account other primary factors such as land and skills. The general conclusion of these studies is that the paradox remained in several cases.¹

Nearly a quarter of a century later, Leamer (1980) provided the definitive critique of the Leontief Paradox. He found that the Leontief Paradox is based on a simple conceptual misunderstanding. While the Leontief Paradox is based on the assumption of balanced trade, the US had a trade surplus in 1947 and was exporting both labor and capital services as embodied in trade. Under trade imbalance, Leamer (1980) theoretically showed that the test employed by Leontief (1953) was not valid, based on the factor content version of the HO model, the so-called Heckscher–Ohlin–Vanek (HOV) model.² He then proposed an alternative test that is robust to trade imbalance and found that, once trade imbalance is taken into account explicitly, US trade patterns in 1947 were consistent with the prediction of the HOV model.³

Shortly after Leamer's finding, Brecher and Choudhri (1982) found another, but closely related, paradox. The fact that the US exported labor services was in itself paradoxical because it can occur if and only if its per-capita consumption is *less than* the world average per-capita consumption. Although this is one of the fun-

¹For more details about the empirical research on the Leontief Paradox from the 1950s to the early 1980s, see Deardorff (1984).

²Because the factor content version of the HO model was developed by Vanek (1968), it is called the Heckscher–Ohlin–Vanek model.

³However, Baldwin (2008) pointed out that "Leamer's reversal of the Leontief paradox failed to hold for other years in the 1950s, 1960s, and 1970s when the US trade balance was adjusted in the manner he specified" (p.86).

damental questions in the HOV model, surprisingly, no formal answer has been provided for nearly four decades. For example, Leamer (1987) argued that "Even after adjusting for trade surplus, this is impossible to square with the facts" (p.2), referring to Brecher and Choudhri's study. Similarly, Davis and Weinstein (2003) stated that "this paradox refused to perish" (p.135). Even more recent textbooks such as Feenstra (2015, p.33) made a similar argument.⁴ Based on the fact that the HOV model remains one of the core models of international trade, it is essential to resolve this paradox.⁵

This paper revisits the paradox outlined by Brecher and Choudhri (1982). The aim of this paper is to present one possible formal solution to the paradox. To do so, I first generalize the paradox from the case of the US to that of any country in Section 2. Then, I investigate the paradox, focusing on five possible explanations: 1) technology differences, 2) trade imbalance, 3) quasi-homothetic preferences, 4) Armington home bias, and 5) offshoring. Here, I show that it is important to distinguish the difference between physically-based consumption and efficiency-based consumption to resolve the paradox. The main results are threefold. First, one can resolve the paradox if the HOV model takes into account technology differences, while trade imbalance is also important for measuring consumption in a precise manner. Second, the paradox cannot be resolved even if the analysis takes into account quasi-homothetic preferences or the Armington home bias. Finally, although offshoring is another logical explanation of the paradox, it is a relatively recent phenomenon and, therefore, is not an explanation of the original paradox. Section 3 discusses the paradox from the view point of the actual data. A summary of the findings is presented in the final section.

⁴Baldwin (2008) and Bowen, Hollander, and Viaene (2012) provided helpful explanations of the debate on the Leontief Paradox.

⁵Although the HOV model is one of the traditional trade models, several studies have improved the HOV model. For recent development of the HOV model, see Ito, Rotunno, and Vézina (2017) and Morrow and Trefler (2017).

2 Theory

2.1 Setup and the paradox

This subsection explains the setup of the standard HOV model and the paradox pointed out by Brecher and Choudhri (1982). Consider many countries, indexed by i = 1, ..., C; many industries, indexed by j = 1, ..., N; and many factors, indexed by k or l = 1, ..., M.⁶ Suppose that countries have identical and homothetic preferences. Assume further that technologies are constant returns scale and identical across countries, and factor price equalization prevails under free trade.⁷. Assume that both goods and factor markets are perfectly competitive, and zero-profit and full-employment conditions hold. Trade costs such as transportation costs and trade barriers are assumed to be negligibly small. Goods are mobile but factors are immobile between countries.

Denote the $M \times N$ matrix $A = [a_{jk}]'$ as the amounts of primary factors such as labor and capital that are needed for one unit of production in each industry, which is common across countries because of the identical technologies. Let Y^i , D^i , and T^i be the vectors of outputs, demands, and net exports of each industry for country *i*, where $T^i = Y^i - D^i$. Country *i*'s GDP is defined as:

$$GDP^{i} = p'Y^{i} = p'D^{i} + p'T^{i},$$
(1)

where p is the price vector and, thus, $p'Y^i$, $p'D^i$, and $p'T^i$ indicate the values of output (or income), consumption, and net exports, respectively.

The factor content of trade is defined as:

$$F^i \equiv AT^i = AY^i - AD^i.$$
⁽²⁾

In this equation, AY^i equals the demand for factors in country *i*. Denote the factor endowment vector in country *i* as V^i . The full employment condition implies:

$$4Y^i = V^i. ag{3}$$

⁶This follows the notation used by Feenstra (2015) for ease of readability of the paper.

⁷Strictly speaking, I assume no factor intensity reversals and incomplete specialization

The identical and homothetic preferences mean that the consumption vectors of all countries are proportional to each other as follows: $D^i = s^i D^W$, where s^i is the share of country *i* in world consumption and D^W denotes the world consumption vector.⁸

Under balanced trade, $p'T^i = 0$. From equation (1), therefore, $GDP^i = p'Y^i = p'D^i$. The share of country *i*'s GDP relative to world GDP is $s^i = p'D^i/p'D^W = p'Y^i/p'Y^W$. Note that, from the market clearing condition, world production must equal world consumption. Thus, $AD^i = A(s^iD^W) = s^i(AY^W) = s^iV^W$, where V^W is the world's factor endowment vector. Therefore:

$$F^i = AY^i - AD^i = V^i - s^i V^W, (4)$$

which is called the HOV equation. In terms of each factor k, this is:

$$F_k^i = V_k^i - s^i V_k^W, (5)$$

which is a statement of the HOV theorem. If country *i*'s endowment of factor *k* relative to the world endowment is greater than country *i*'s share of world GDP (i.e., $V_k^i/V_k^W > s^i$ or $V_k^i > s^iV_k^W$), then it is said that country *i* is abundant in that factor and, thus, the factor-content of trade in factor *k* will be positive, $F_k^i > 0$, and vice versa for the scarce factor.

Focusing on two factors, capital *K* and labor *L*, equation (5) is written as:

$$F_K^i = K^i - s^i K^W$$
 and $F_L^i = L^i - s^i L^W$. (6)

While Leamer (1980) proposed and conducted an alternative test for the Leontief Paradox based on equation (6), he also found that the US (i = US) had a trade surplus in 1947 and was exporting both capital and labor as embodied in trade $F_K^{US} > 0$ and $F_L^{US} > 0$, respectively.

From equation (6), F_L^i is rewritten as:

$$F_L^i = \frac{L^i L^W}{Y^W} \left(\frac{Y^W}{L^W} - \frac{Y^i}{L^i} \right). \tag{7}$$

⁸Hereafter, superscript W indicates the world total (e.g., $D^W = \sum_j^C D^j$).

From $L^i, L^W, Y^W > 0$, I have:

$$\operatorname{sign}(F_L^i) = \operatorname{sign}\left(\frac{Y^W}{L^W} - \frac{Y^i}{L^i}\right).$$
(8)

Therefore, $Y^W/L^W - Y^i/L^i > 0$ means that country *i*'s per-capita consumption is *below* the world average per-capita consumption.

In this context, based on the fact that the US (i = US) was exporting labor as embodied in trade in 1947 (i.e., $F_L^{US} > 0$), Brecher and Choudhri (1982) pointed out that:

$$F_L^{US} > 0 \quad \Leftrightarrow \quad \frac{Y^W}{L^W} > \frac{Y^{US}}{L^{US}},$$
(9)

which contradicts the actual data: $Y^W/L^W < Y^{US}/L^{US}$. In words, Brecher and Choudhri (1982) argued that the fact that the US exported labor services (i.e., $F_L^{US} > 0$) was in itself paradoxical because US per-capita consumption is above the world average per-capita consumption (i.e., $Y^W/L^W < Y^{US}/L^{US}$). Brecher and Choudhri (1982) concluded that "the Leontief Paradox is still with us" as quoted at the beginning of this article.

Note that their discussion is based on the comparison between a country's percapita consumption and the world average per-capita consumption. In a similar fashion, it is also paradoxical if $F_L^i < 0$ and country *i*'s per-capita consumption is below the world average per-capita consumption (i.e., $Y^W/L^W > Y^i/L^i$) because the HOV equation suggests $F_L^i < 0 \Leftrightarrow Y^W/L^W - Y^i/L^i < 0$. Noting that the paradox pointed out by Brecher and Choudhri (1982) comes from the difference in sign, it can be generalized as follows:

Paradox. It is paradoxical if the actual data indicate:

$$sign(F_L^i) \neq sign\left(\frac{Y^W}{L^W} - \frac{Y^i}{L^i}\right),$$
 (10)

because equation (10) contradicts the sign of the HOV equation (i.e., equation (8)).

This is a general statement of the paradox.

2.2 Technology differences

One of the restrictive assumptions in the HOV model is that technology is common to all countries.⁹ One way to relax this assumption was developed by Trefler (1993), which allows all factors in every country to differ in their productivities. This adjustment is important because it allows me to measure endowments in terms of efficiency units rather than physical units.

Let π_k^i $(0 < \pi_k^i)$ be the productivity of factor k in country i relative to its productivity in the US ($\pi_k^{US} = 1$). In terms of efficiency units, the effective endowment of factor k in country i is π_k^i . The HOV equation is rewritten in terms of effective factor endowments as follows:

$$F_{k}^{i} = \pi_{k}^{i} V_{k}^{i} - s^{i} \sum_{j=1}^{C} \pi_{k}^{j} V_{k}^{j} = \tilde{V}_{k}^{i} - s^{i} \tilde{V}_{k}^{W},$$
(11)

where $\tilde{V}_k^i (= \pi_k^i V_k^i)$ and $\tilde{V}_k^W (= \sum_{j=1}^C \pi_k^j V_k^j)$ are the effective factor endowments of country *i* and the world, respectively.

Note that F_L^i is rewritten in terms of efficiency units as follows:

$$F_L^i = \frac{\tilde{L}^i \tilde{L}^W}{Y^W} \left(\frac{Y^W}{\tilde{L}^W} - \frac{Y^i}{\tilde{L}^i} \right).$$
(12)

Because $\tilde{L}^i, \tilde{L}^W, Y^W > 0$, I have:

$$\operatorname{sign}(F_L^i) = \operatorname{sign}\left(\frac{Y^W}{\tilde{L}^W} - \frac{Y^i}{\tilde{L}^i}\right).$$
(13)

This equation implies that, if $F_L^i > 0$, country *i*'s per-efficiency-unit consumption is *below* the world average per-efficiency-unit consumption: $Y^W/\tilde{L}^W > Y^i/\tilde{L}^i$. Therefore:¹⁰

$$F_L^i > 0 \quad \Leftrightarrow \quad \frac{Y^W}{\tilde{L}^W} > \frac{Y^i}{\tilde{L}^i}.$$
 (14)

Equation (14) is similar to equation (9) but has a different implication. Equation (14) could be consistent with the case when country i's per-capita consumption

⁹In this connection, Helpman (1999) argued that "allowing for differences in techniques of production can dramatically improve the fit of factor content equations" (p.133).

¹⁰Equation (14) is rewritten: $\tilde{L}^i > s^i \tilde{L}^W$. Equation (14) thus, simply states that country *i* is a labor abundant country in terms of efficiency units.

is greater than the world average per-capita consumption: $Y^W/L^W < Y^i/L^{i.11}$ This is because the paradox comes from physically-based consumption, while the HOV equation is now built upon efficiency-based consumption. In other words, if productivity differences are introduced in this manner, the analysis can distinguish efficiency-based consumption from physically-based consumption.

To see this point, let me assume that productivity is the same for all countries except country *i*: $\pi_k^j = \pi_k^{-i}$, $\forall j \neq i$. Equation (11) can be rewritten as:

$$F_L^i = \pi_L^i L^i - s^i \sum_{j=1}^C \pi_L^{-i} L^j = \pi_L^i L^i - s^i \pi_L^{-i} L^W.$$
(15)

If $F_L^i > 0$, $\pi_L^i L^i - s^i \pi_L^{-i} L^W > 0$. Therefore:

$$\pi_L^i L^i > s^i \pi_L^{-i} L^W.$$
(16)

Noting that $s_i = p'D^i/p'D^W (= p'Y^i/p'Y^W)$, I have:¹²

$$\frac{Y^W}{\pi_L^{-i}L^W} > \frac{Y^i}{\pi_L^i L^i}.$$
(17)

Unlike the standard HOV equation, this equation could be consistent with the case in which country *i*'s per-capita consumption is greater than the world average percapita consumption.

Let me give a simple numerical example. Suppose that country *i*'s per-capita consumption is twice that of the world average: $Y^W/L^W = 1 < 2 = Y^i/L^i$. However, if country *i*'s productivity is three times as large as the world average's productivity: $\pi_L^i = 1 > \pi_L^{-i} = 1/3$, I have:

$$\frac{Y^W}{\pi_L^{-i}L^W} = 3 > 2 = \frac{Y^i}{\pi_L^i L^i}.$$
(18)

Because both $\operatorname{sign}(F_L^i) \neq \operatorname{sign}(Y^W/L^W - Y^i/L^i)$ and $\operatorname{sign}(F_L^i) = \operatorname{sign}(Y^W/\tilde{L}^W - V^i/L^i)$

¹¹When $F_L^i > 0$ and $Y^W/L^W < Y^i/L^i$, from equation (14), I have $\tilde{L}^i/\tilde{L}^W > Y^i/Y^W > L^i/L^W$. This relationship is not surprising when the productivity of country *i* is greater than that of other countries.

¹²In this particular case, equation (17) can be rewritten as: $\pi_L^i/\pi_L^{-i} > (Y^i/L^i)/(Y^W/L^W)$. This means that country *i*'s relative productivity is greater than its relative per-capita consumption.

 Y^i/\tilde{L}^i) hold simultaneously, the sign of relative per-capita consumption does not contradict the HOV equation. That is, the paradoxical situation disappears.

For a more general case when productivities differ across countries, Trefler (1993) derived the following theorem.

Theorem. Allowing for all factors in all but one country to differ in their productivities π_k^i , there will be a solution for productivities π_k^i such that equation (12) holds with equality for i = 1, ..., C and k = 1, ..., M for almost all datasets.

Proof. See Trefler (1993).

This in turn implies that equation (13) holds under certain values of productivities. Noting that the paradox pointed out by Brecher and Choudhri (1982) comes from relative per-capita consumption in physical units rather than efficiency units, I obtain the following proposition.

Proposition. If one can find productivity values that equalize F_L^i and $\pi_L^i L - s^i \sum_{j=1}^C \pi_L^j L^j$, the paradox outlined by Brecher and Choudhri (1982) is resolved.

Proof. Under the productivity values that equalize F_L^i and $\pi_L^i L^i - s^i \sum_{j=1}^C \pi_L^j L^j$, equation (13) holds, regardless of the relationship between $\operatorname{sign}(F_L^i)$ and $\operatorname{sign}(Y^W/L^W - Y^i/L^i)$. This in turn means that $\operatorname{sign}(F_L^i) \neq \operatorname{sign}(Y^W/L^W - Y^i/L^i)$ does not contradict the HOV equation. Therefore, the paradox no longer exists.

This proposition states that the paradox is resolved if equation (13) holds under equation (10). The proposition suggests that, even if the US was a net exporter of labor services in 1947, it does not necessarily contradict the HOV equation when the endowment is measured in efficiency units. Empirically, checking whether equation (13) holds or not is the same as the sign test employed by Bowen, Leamer, and Sveikauskas (1987). However, the test has an additional restriction as in equation (10). This means that the usual sign test is not adequate in investigating the paradox outlined by Brecher and Choudhri (1982) because it includes both paradoxical and nonparadoxical cases. In other words, in order to resolve the paradox, the sign test of equation (13) should be conducted, conditional on equation (10). In this context, Trefler (1993) estimated the productivity parameters that satisfy the HOV equation as an identity.¹³ This in turn means that, under the productivity estimates by Trefler (1993), the paradox outlined by Brecher and Choudhri (1982) is resolved. One may then ask whether relaxing other assumptions of the HO model can also resolve the paradox. To address this issue, the following subsections investigate four other possibilities: trade imbalance, quasi-homothetic preferences, Armington home bias, and offshoring.

2.3 Trade imbalance

Many standard trade models assume balanced trade because the models are generally static and, thus, focus on long-run equilibrium. However, the results and testable implications sometimes change if one relaxes this assumption, as pointed out by Leamer (1980).¹⁴ Therefore, it is important to note the existence of trade imbalance when one investigates the empirical validity of trade theories.

When trade is unbalanced, $p'T^i \neq 0$. Let the consumption share under trade imbalance be s^{*i} . Under trade imbalance, the consumption share s^{*i} is:¹⁵

$$s^{*i} = \frac{p'D^i}{p'D^W} = \frac{Y^i - T^i}{Y^W}.$$
(19)

Under trade imbalance, F_L^i is:

$$F_{L}^{i} = \frac{L^{i}L^{W}}{Y^{W}} \left(\frac{Y^{W}}{L^{W}} - \frac{Y^{i} - T^{i}}{L^{i}}\right).$$
 (20)

From $L^i, L^W, Y^W > 0$, I have:

$$\operatorname{sign}(F_L^i) = \operatorname{sign}\left(\frac{Y^W}{L^W} - \frac{Y^i - T^i}{L^i}\right).$$
(21)

¹³However, Trefler's (1993) productivity estimates are controversial. For example, Gabaix (1997) pointed out that the productivity parameters would be mechanically correlated with wages if wages are correlated with GDP per capita. For more details, see Davis and Weinstein (2003).

¹⁴Kiyota (2011) also pointed out that the test of the law of comparative advantage by Bernhofen and Brown (2004) is applicable only to the case of balanced trade, and proposed a more general test that is consistent with both balanced and unbalanced trade.

¹⁵If Y^W covers all countries, the sum of trade balances will be zero from the market clearing condition: $\sum_{j=1}^{C} p'T^j = 0$ and thus $\sum_{j=1}^{C} (p'Y^j - p'T^j) = \sum_{j=1}^{C} p'Y^j = p'Y^W$. Therefore, $p'D^W = p'Y^W$. See, also, Trefler (1993, p.964).

As before, $Y^W/L^W - (Y^i - T^i)/L^i > 0$ means that country *i*'s per-capita consumption (excluding net exports) is *below* world average per-capita consumption: $Y^W/L^W > (Y^i - T^i)/L^i$. This in turn implies:

$$F_L^i > 0 \quad \Leftrightarrow \quad \frac{Y^W}{L^W} > \frac{Y^i - T^i}{L^i}.$$
 (22)

The implication under trade imbalance remains essentially the same as under trade balance. However, equation (22) indicates that some adjustments are required to measure per-capita consumption correctly when trade is unbalanced because per-capita consumption is not Y^i/L^i but $(Y^i - T^i)/L^i$.¹⁶

2.4 Quasi-homothetic preferences

Another restrictive assumption of the HOV model is that preferences are homothetic across countries. For example, based on an empirical model with quasihomothetic preferences, Hunter and Markusen (1988) found statistically significant nonhomothetic preferences in world demand patterns.¹⁷ To address this issue, I follow the model developed by Torstensson (1993). In Torstensson's model, preferences are nonhomothetic and there are two factors (i.e., capital *K* and labor *L*) of production. All other assumptions are retained as in the standard HOV model, including balanced trade and identical constant returns-to-scale technology.

Assume that all individuals have identical and quasi-homothetic preferences. They first have to consume a given subsistence bundle of goods. The remaining income is allocated proportionally across industries.¹⁸ Therefore, both production and consumption are divided into subsistence and nonsubsistence. The factor content of trade equation (2) becomes:¹⁹

$$AT^{i} = AY^{i} - A(D_{0}^{i} + D_{1}^{i}), (23)$$

¹⁶Trefler and Zhu (2010) also argued that it is important to employ such an adjustment in computing the country's share of world consumption in testing the prediction of the HOV model.

¹⁷In quasi-homothetic preferences, the Engel curves are linear but not through the origin. This is called a linear expenditure system. Recent studies also argued the importance of nonhomothetic preferences in explaining bilateral trade patterns. See, for example, Fieler (2011).

¹⁸Without loss of generality, I assume that the income is large enough to cover both subsistence and nonsubsistence consumption.

¹⁹If there is no subsistence, equation (23) reverts to equation (2).

where D_0^i and D_1^i are the vectors of subsistence and nonsubsistence consumption, respectively. Because all individuals have identical subsistence consumption, country *i*'s share of subsistence consumption equals its share of world population:

$$D_0^i = \frac{L^i}{L^W} Y_0^W, \tag{24}$$

where Y_0^W is the vector of world subsistence production. In contrast, quasi-homothetic preferences mean that country *i*'s vector of nonsubsistence consumption is proportional to its share of world nonsubsistence income, s_1^i :

$$D_1^i = s_1^i Y_1^W, (25)$$

where Y_1^W is the vector of world nonsubsistence production and $s_1^i = Y_1^i/Y_1^W$.

Let V^W , V_0^W , and V_1^W be the vectors of world factor endowment used for total production, subsistence production, and nonsubsistence production, respectively: $V^W = V_0^W + V_1^W$. The full employment condition is:

$$AY_0^W = V_0^W$$
 and $AY_1^W = V_1^W$. (26)

From equations (23)–(26), the factor content of trade becomes:

$$AT^{i} = V^{i} - \left(\frac{L^{i}}{L^{W}}\right) V_{0}^{W} - s_{1}^{i} V_{1}^{W}.$$
(27)

Let μ_L (0 < μ_L < 1) be the share of total world endowment of labor used for subsistence production: $\mu_L = L_0^W / L^W$. For labor inputs, equation (27) becomes:

$$F_{L}^{i} = L_{i} - \left(\frac{L^{i}}{L^{W}}\right) \mu_{L} L^{W} - s_{1}^{i} (1 - \mu_{L}) L^{W}$$

= $(1 - \mu_{L}) (L^{i} - s_{1}^{i} L^{W}).$ (28)

This equation is then:

$$F_L^i = (1 - \mu_L) \frac{L^i L^W}{Y_1^W} \left(\frac{Y_1^W}{L^W} - \frac{Y_1^i}{L^i} \right).$$
⁽²⁹⁾

Because $0 < \mu_L < 1$ and $L^W, L^W, Y_1^W > 0$, I have:

$$\operatorname{sign}(F_L^i) = \operatorname{sign}\left(\frac{Y_1^W}{L^W} - \frac{Y_1^i}{L^i}\right). \tag{30}$$

As for the previous discussions, $Y_1^W/L^W - Y_1^i/L^i > 0$ means that country *i*'s percapita consumption of nonsubsistence goods is *below* world average per-capita consumption: $Y_1^W/L^W > Y_1^i/L^i$. Therefore:

$$F_L^i > 0 \quad \Leftrightarrow \quad \frac{Y_1^W}{L^W} > \frac{Y_1^i}{L^i}.$$
 (31)

Equation (31) states that, if country i is a net exporter of labor, its per-capita consumption of nonsubsistence goods is less than the world average. The only difference between equations (9) and (31) is whether consumption is of all goods or of only nonsubsistence goods. When country i is the US, this equation states that the per-capita US consumption of nonsubsistence goods is lower than the world average consumption, which is clearly unrealistic. This in turn implies that, while quasi-linear preferences may be more realistic than homothetic preferences, the paradox cannot be resolved even when quasi-homothetic preferences are employed.

2.5 Armington home bias

One may be concerned that Armington home bias is another possible way of resolving the paradox. To address this concern, this subsection introduces Armington home bias into the baseline framework in Section 2. Following Trefler (1995), from $D^i = s^i D^W = s^i Y^W$, I distinguish between domestic and foreign goods as follows:

$$D^{i} = s^{i} \left[\alpha^{i} Y^{i} + \beta^{i} \left(Y^{W} - Y^{i} \right) \right], \qquad (32)$$

where $\beta^i < 1 < \alpha^i$ captures home bias.²⁰

Without measurement error, the variant of the HOV equation (6) for labor im-

²⁰Theoretically, β^i can be negative. For example, if $\alpha^i = 2.5$ and $\beta^i = -0.5$, $Y^i/Y^W = (1 - \beta^i)/(\alpha^i - \beta^i) = 1/2$.

plied by the Armington assumption is:

$$F_L^i = L^i - \frac{Y^i}{Y^W} \left[(1 - \beta^i) \frac{Y^W}{Y^i} L^i + \beta^i L^W \right]$$
(33)

$$= \beta^{i} \frac{L^{i} L^{W}}{Y^{W}} \left(\frac{Y^{W}}{L^{W}} - \frac{Y^{i}}{L^{i}} \right).$$
(34)

So long as $\beta^i > 0$, I have:

$$\operatorname{sign}(F_L^i) = \operatorname{sign}\left(\frac{Y^W}{L^W} - \frac{Y^i}{L^i}\right),\tag{35}$$

which is the same as equation (8).

Another possible explanation of the paradox is $\beta^i < 0$. However, as Trefler (1995) argued, it is difficult to interpret negative β^i . Besides, he confirmed only three out of 33 countries had a significantly negative β^i using actual data. Therefore, a negative β^i does not seem to explain the paradox. This in turn means that the paradox cannot be resolved even if Armington home bias is introduced, although it is important in explaining the missing trade.

2.6 Offshoring

One may be further concerned about the role of offshoring. As technology differences play a prominent role in improving the fit of factor content equations (Helpman, 1999), whether or not a country offshores, which has many similar features to technology differences, may have a similar effect. Offshoring has two aspects. The first is international factor movement and the second is increases in trade in intermediate inputs through the fragmentation of production process. The following subsections examine each of these aspects.

2.6.1 International factor movement

To address the issue of international factor movement in the HOV model, I follow the framework developed by Gaisford (1995). Suppose that a subset of factors can be mobile between countries without cost while maintaining the other assumptions. Let $V_k^{h,i}$ be the vector of factors deployed in the home country. Let $V_k^{f,i}$ be the vector of factors moved between countries in net terms. Therefore, $V_k^{f,i} > 0$ and $V_k^{f,i} < 0$ mean a net outflow and net inflow of factor k in country i, respectively. Define the vector of total factor endowments of country i as: $V_k^i \equiv V_k^{h,i} + V_k^{f,i}$. Equation (5) can be rewritten as:

$$F_k^i = V_k^{h,i} - s^i V_k^W$$
 or $F_k^i + V_k^{f,i} = V_k^i - s^i V_k^W$. (36)

The left-hand-side of the HOV equation now includes the factors that move between countries, while V_k^i on the right-hand-side indicates the sum of factors at home and abroad. For example, if some of the capital stocks are allocated abroad through foreign direct investment, then $K^{f,i} > 0$. This HOV equation implies that a capital-abundant country could have negative net exports of capital (i.e., $F_K^i < 0$) if capital stocks deployed abroad ($K^{f,i}$) are large. Similarly for labor input, if the home country consumes foreign labor services directly through services offshoring, $L^{f,i} < 0$.²¹ A labor-abundant country could have negative net exports of labor (i.e., $F_L^i < 0$) if labor services deployed abroad ($L^{f,i}$) are large.

When capital is mobile while labor is immobile internationally, Gaisford (1995) found that the US exports of goods tended to be labor intensive because of US capital stocks deployed abroad (i.e., $K^{f,i}$). Note, however, that when labor is immobile between countries, no labor is deployed in foreign countries: $L^{f,i} = 0$. From $L^i \equiv L^{f,i} + L^{h,i}$, $L^{f,i} = 0$ means $L^i = L^{h,i}$. Therefore, the HOV equation for labor is not affected and, thus, the paradox cannot be resolved even when the analysis allows for capital movement between countries if labor is immobile.

When labor is mobile between countries, some of the labor services could be deployed abroad. Equation (36) for labor is written as:

$$F_L^i = \frac{L^i L^W}{Y^W} \left(\frac{Y^W}{L^W} - \frac{Y^i}{L^i}\right) - L^{f,i}.$$
(37)

This in turn implies that the different signs of F_L^i and $(Y^W/L^W - Y^i/L^i)$ do not necessarily mean the paradoxical situation because of labor services deployed abroad (i.e., $L^{f,i}$). Therefore, the paradox outlined by Brecher and Choudhri (1982) seems to be resolved if the analysis takes into account international labor movement.

²¹Head, Mayer, and Ries (2009) argued that "the key idea of service offshoring is that a firm can replace the services of domestic workers *directly* with the services of workers residing in foreign countries ("offshore"). Foreign workers can supply their services via communication or via temporary visits to the domestic producer's facility" (p.434).

Note, however, that if the US exports labor services (i.e., $F_L^{US} > 0$), US percapita consumption is above the world average per-capita consumption (i.e., $Y^W/L^W < Y^{US}/L^{US}$) only when the net outflow of US labor services is sufficiently large. This may not be plausible for the following two reasons. First, international labor movement was costly in 1947 when the paradox was confirmed. Second, the outflow of labor services needs to exceed the inflow of labor services in the US, even though the US has accepted large numbers of immigrants historically. While international labor movement could be one logical explanation for the paradox, it does not seem to be a plausible explanation in reality. Therefore, a simple international factor movement does not seem to resolve the paradox.

2.6.2 Trade in intermediate inputs

In the discussion on offshoring above, I maintain the assumption that factor price equalization prevails under free trade. When offshoring involves the fragmentation of production processes, one needs to relax this assumption so that the endowments of countries can locate in the different cones of diversification (Deardorff, 2001).²² Indeed, Schott (2003) and Kiyota (2012) presented empirical evidences for the existence of multiple cones. Besides, the fragmentation of production processes leads to increases in trade in intermediate inputs.²³

Trefler and Zhu (2010) showed that the HOV model could be consistent with both international technology differences and trade in intermediate inputs. With the assumption that each country's consumption of any other country's good is a fixed proportion of world consumption for that good, the choice of production techniques can vary across countries. Their study argued that the HOV equation (5) holds even when the HOV model accounts for both international technology differences and trade in intermediate inputs. However, the measurement of factor content of trade is different. The analysis requires knowledge not only of what intermediate inputs are used in the production of goods, but also of where these inputs come from. Their empirical analysis is, thus, built upon a world inputoutput table.

²²In Section 2.2, Trefler (1993) also assumed productivity-equivalent factor price equalization under technology differences.

²³For example, Kimura (2006) pointed out that active back-and-forth transactions of machinery parts and components are observed among countries with different income levels in East Asia.

In the context of my study, exported labor services from the US in 1947 may have been estimated incorrectly because the analysis did not take into account international technology differences and trade in intermediate inputs. In order to take these two issues into account, however, a world input–output table is needed, which has only been available in recent years. Besides, fragmentation of the production process is a relatively recent phenomenon. While fragmentation through offshoring is another logical explanation of the paradox, it does not seem to be the main reason for the paradox.

In sum, offshoring is another logical explanation of the paradox. However, the growth of offshoring is a relatively recent phenomenon. Because simple international factor movement does not seem to explain the paradox, it is not necessarily clear whether offshoring was regarded as an important factor when the paradox was confirmed.

3 Discussion

3.1 Does the paradox matter in reality?

Section 1 argued that no formal answer has been provided for nearly four decades. A possible reason for this is that the US has not been a net exporter of labor services since the 1960s.²⁴ A concern is that, although the paradox may matter theoretically, it does not matter in reality.

From equation (21), under trade imbalance, the paradox stems from:

$$\operatorname{sign}(F_L^i) \neq \operatorname{sign}\left(\frac{Y^W}{L^W} - \frac{Y^i - T^i}{L^i}\right).$$
(38)

We check these signs, using the data used in Trefler (1993) because of its reliability and accessibility.²⁵ Another advantage of the use of his data is that, because the

²⁴See, for example, Bowen, Leamer, and Sveikauskas (1987, Table 1) for the year 1967, and Maskus (1985, Table 2) for the year 1972.

²⁵The data are obtained from the website for Feenstra's (2015) book. Because Trefler's data do not cover all countries, the sum of net exports is not zero (i.e., $\sum_j p'T^j \neq 0$). Therefore, $Y^W \neq D^W$. Therefore, I also adjust world average consumption: $D^W = Y^W - T^W$ to compute $D^W/L^W - (Y^i - T^i)/L^i$ and $D^W/\tilde{L}^W - (Y^i - T^i)/\tilde{L}^i$. However, I continue to use Y^W instead of D^W for ease of

data are based on the year 1983, offshoring was still not common and, thus, it is plausible to assume that its effect is small.

Table 1 presents the results. I highlight two main findings. First, the US was a net importer of labor services in 1983 (i.e., $F_L^{US} < 0$). Its per-capita consumption was also higher than the world average (i.e., $Y^W/L^W - (Y^{US} - T^{US})/L^{US} > 0$). Because the signs are the same, no paradox was confirmed for the US in 1983. Second, sign $(F_L^i) \neq \text{sign}(Y^W/L^W - (Y^i - T^i)/L^i)$ is confirmed for 18 out of 33 countries. For example, Japan and West Germany experienced the paradox in 1983 because $F_L^i > 0$ and $Y^W/L^W - (Y^i - T^i)/L^i < 0$, which is similar to the case of the US in 1947. According to Brecher and Choudhri (1982), these 18 countries experienced the paradox. Using more recent data, Kiyota (2013) confirmed that Japan experienced the paradox in 17 of the 26 years from 1980 to 2005. These results together imply that the paradox matters in reality.

3.2 Does trade imbalance really matter?

One may ask whether trade imbalance really matters because Section 2 indicates that the implications under trade imbalance remains essentially the same as those under trade balance. To answer this question, I investigate whether the paradox can be resolved without adjusting the trade imbalance, while taking into account the productivity difference only.

Table 2 indicates that, without adjusting the trade imbalance, the paradox still exists for 14 of 33 countries. Note that under Trefler's (1993) productivity estimates, the paradox is completely resolved once the trade imbalance is adjusted because he estimated productivity parameters such that the HOV equation can hold as an identity. These results suggest that adjustment of the trade imbalance also matters.

3.3 Alternative productivity estimates

One may also ask whether the paradox can be resolved if we use alternative productivity estimates. In this context, Trefler (1995) estimated country-specific productivity parameters π^i instead of country-factor-specific productivity parameters

exposition.

$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Country	F_L^i	Y^W/L^W	Paradox
Bangladesh $-11,939$ $19,107$ YesBelgium $-54,717$ $-8,359$ Canada $375,879$ $-8,489$ YesColombia $-31,908$ $12,153$ YesDenmark $96,426$ $-5,512$ YesFinland $87,115$ $-5,190$ YesFrance $-126,439$ $-8,367$ GreeceGreece $-100,890$ $6,683$ YesHong Kong $28,988$ $5,823$ IndonesiaIndonesia $-21,490$ $18,091$ YesIreland $13,208$ $4,336$ Israel $-62,401$ $-5,488$ Italy $353,051$ $-2,030$ YesJapan $1,037,299$ $-2,498$ YesNetherlands $149,314$ $-9,230$ YesNorway $-8,048$ $-10,121$ PakistanPakistan $-7,402$ $18,245$ YesPortugal $-34,732$ $12,709$ YesSingapore $-136,613$ $-3,975$ SpainSpain $-51,474$ $3,666$ YesSri Lanka $-11,4411$ $18,337$ YesSweden $107,829$ $-6,819$ YesSwitzerland $-112,607$ $-17,096$ Thailand $20,023$ $17,922$ Trinidad $-47,088$ $-2,476$ UK $-638,258$ $-5,495$ Uruguay $23,905$ $12,922$ USA $-1,570,669$ $-18,816$ West Germany $467,201$ $-8,723$ Yes			$-(Y^i - T^i)/L^i$	
Belgium $-54,717$ $-8,359$ Canada $375,879$ $-8,489$ YesColombia $-31,908$ $12,153$ YesDenmark $96,426$ $-5,512$ YesFinland $87,115$ $-5,190$ YesFrance $-126,439$ $-8,367$ GreeceGreece $-100,890$ $6,683$ YesHong Kong $28,988$ $5,823$ IndonesiaIndonesia $-21,490$ $18,091$ YesIreland $13,208$ $4,336$ Israel $-62,401$ $-5,488$ Italy $353,051$ $-2,030$ YesJapan $1,037,299$ $-2,498$ YesNetherlands $149,314$ $-9,230$ YesNew Zealand $58,226$ -734 YesNorway $-8,048$ $-10,121$ PakistanPakistan $-7,402$ $18,245$ YesPortugal $-34,732$ $12,709$ YesSingapore $-136,613$ $-3,975$ SpainSpain $-51,474$ $3,666$ YesSri Lanka $-11,441$ $18,337$ YesSweden $107,829$ $-6,819$ YesSwitzerland $-112,607$ $-17,096$ Thailand $20,023$ $17,922$ Trinidad $-47,088$ $-2,476$ UK $-638,258$ $-5,495$ Uruguay $23,905$ $12,922$ USA $-1,570,669$ $-18,816$ West Germany $467,201$ $-8,723$ Yes	Austria	-71,962	-2,716	
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New Zealand58,226-734YesNorway-8,048-10,121Pakistan-7,40218,245YesPanama-19,4209,670YesPortugal-34,73212,709YesSingapore-136,613-3,975SpainSpain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Japan	1,037,299	-2,498	Yes
Norway-8,048-10,121Pakistan-7,40218,245YesPanama-19,4209,670YesPortugal-34,73212,709YesSingapore-136,613-3,975SpainSpain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Netherlands	149,314	-9,230	Yes
Pakistan-7,40218,245YesPanama-19,4209,670YesPortugal-34,73212,709YesSingapore-136,613-3,975Spain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	New Zealand	58,226	-734	Yes
Panama-19,4209,670YesPortugal-34,73212,709YesSingapore-136,613-3,975Spain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Norway	-8,048	-10,121	
Portugal-34,73212,709YesSingapore-136,613-3,975Spain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Pakistan	-7,402	18,245	Yes
Singapore-136,613-3,975Spain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Panama	-19,420	9,670	Yes
Spain-51,4743,666YesSri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Portugal	-34,732	12,709	Yes
Sri Lanka-11,44118,337YesSweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Singapore	-136,613	-3,975	
Sweden107,829-6,819YesSwitzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Spain	-51,474	3,666	Yes
Switzerland-112,607-17,096Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Sri Lanka	-11,441	18,337	Yes
Thailand20,02317,922Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Sweden	107,829	-6,819	Yes
Trinidad-47,088-2,476UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Switzerland	-112,607	-17,096	
UK-638,258-5,495Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Thailand	20,023	17,922	
Uruguay23,90512,922USA-1,570,669-18,816West Germany467,201-8,723Yes	Trinidad	-47,088	-2,476	
USA -1,570,669 -18,816 West Germany 467,201 -8,723 Yes	UK	-638,258	-5,495	
USA -1,570,669 -18,816 West Germany 467,201 -8,723 Yes	Uruguay	23,905	12,922	
	USA	-1,570,669	-18,816	
	West Germany	467,201	-8,723	Yes
		36,406	12,326	

Table 1: The Paradox Outlined by Brecher and Choudhri (1982)

Note: The paradox = "Yes" if sign $(F_L^i) \neq$ sign $\left(\frac{Y^W}{L^W} - \frac{Y^i - T^i}{L^i}\right)$. Source: Computed from Trefler's (1993) data.

	F_L^i	Y^W/L^W	Paradox
		$-Y^i/L^i$	
Austria	-71,962	194	Yes
Bangladesh	-11,939	2,304	Yes
Belgium	-54,717	-464	
Canada	375 <i>,</i> 879	518	
Colombia	-31,908	216	Yes
Denmark	96,426	2,157	
Finland	87,115	2,019	
France	-126,439	265	Yes
Greece	-100,890	764	Yes
Hong Kong	28,988	2,942	
Indonesia	-21,490	-1,276	
Ireland	13,208	1,703	
Israel	-62,401	2,221	Yes
Italy	353,051	1,187	
Japan	1,037,299	136	
Netherlands	149,314	-12	Yes
New Zealand	58,226	2,642	
Norway	-8,048	-3,803	
Pakistan	-7,402	1,891	Yes
Panama	-19,420	2,706	Yes
Portugal	-34,732	3,149	Yes
Singapore	-136,613	1,908	Yes
Spain	-51,474	917	Yes
Sri Lanka	-11,441	1,612	Yes
Sweden	107,829	578	
Switzerland	-112,607	-658	
Thailand	20,023	3,463	
Trinidad	-47,088	-10,513	
UK	-638,258	-1,709	
Uruguay	23,905	2,674	
USA	-1,570,669	-223	
West Germany	467,201	-342	Yes
Yugoslavia	36,406	1,737	

Table 2: The Paradox without the Adjustment of Trade Imbalance

Note: The paradox = "Yes" if sign $(F_L^i) \neq$ sign $\left(\frac{Y^W}{L^W} - \frac{Y^i}{L^i}\right)$. Source: Computed from Trefler's (1993) data. π_L^i . This subsection investigates how the results change if I use country-specific productivity parameters.

I first check the correlation between the country-factor-specific productivity estimates π_L^i and the country-specific productivity estimates π^i . The correlation is high at 0.950. At first sight, this high correlation suggests that alternative productivity estimates also work well. The results in Table 3, however, indicate that 12 of 33 countries are experiencing the paradox. These results together imply that the use of the country-specific productivity estimates cannot resolve the paradox. The results suggest that whether or not the paradox can be resolved empirically depends upon the estimates of productivity.²⁶

4 Concluding Remarks

Shortly after Leamer (1980) found that the Leontief Paradox was based on a simple conceptual misunderstanding, Brecher and Choudhri (1982) found another, but closely related, paradox. The fact that the US exported labor services was in itself paradoxical because it is true if and only if its per-capita consumption is less than the world average per-capita consumption. Even though this is one of the fundamental questions in the HOV model, surprisingly, no formal answer has been provided for nearly four decades.

This paper revisits the paradox outlined by Brecher and Choudhri (1982). Building upon the study by Trefler (1993), I showed that the paradox can be resolved if the HOV model takes into account technology differences, while trade imbalance is also important to the precise measurement of consumption. This is because the introduction of technology differences enables one to distinguish between physically-based and efficiency-based consumption. Besides, a country's consumption is under- or over-estimated without the adjustment of the trade imbalance. In addition, this paper showed that the paradox cannot be resolved even if the analysis takes into account quasi-homothetic preferences, or Armington home bias. While fragmentation through offshoring is another logical way to resolve the paradox, it does not seem to be the main reason when the paradox was found

²⁶In this context, Fisher and Marshall (2016) pointed out the problem of only using the US technology matrix. However, as mentioned above, Trefler and Zhu (2010) had already presented a more general framework that relaxed this assumption.

Country	F_L^i	$Y^W/\pi^i L^W$	Paradox
	- L	$-(Y^i - T^i)/\pi^i L^i$	1 01000000
Austria	-71,962	-843	
Bangladesh	-11,939	15,079	Yes
Belgium	-54,717	-6,644	100
Canada	375,879	-14,744	Yes
Colombia	-31,908	-10,890	100
Denmark	96,426	1,996	
Finland	87,115	-1,768	Yes
France	-126,439	-1,395	100
Greece	-100,890	7,561	Yes
Hong Kong	28,988	1,780	100
Indonesia	-21,490	19,991	Yes
Ireland	13,208	8,575	100
Israel	-62,401	-14,907	
Italy	353,051	300	
Japan	1,037,299	4,818	
Netherlands	149,314	-3,649	Yes
New Zealand	58,226	-17,306	Yes
Norway	-8,048	-6,690	
Pakistan	-7,402	19,856	Yes
Panama	-19,420	595	Yes
Portugal	-34,732	-13,703	
Singapore	-136,613	-12,827	
Spain	-51,474	-1,697	
Sri Lanka	-11,441	20,885	Yes
Sweden	107,829	-10,011	Yes
Switzerland	-112,607	-10,039	
Thailand	20,023	25,835	
Trinidad	-47,088	-10,689	
UK	-638,258	-6,926	
Uruguay	23,905	-25,492	Yes
USA	-1,570,669	-1,963	
West Germany	467,201	97	
Yugoslavia	36,406	10,995	

Table 3: The Paradox under Alternative Productivity Estimates

Note: The paradox = "Yes" if sign $(F_L^i) \neq$ sign $\left(\frac{Y^W}{\pi^i L^W} - \frac{Y^i - T^i}{\pi^i L^i}\right)$. Source: Computed from Trefler's (1993) and (1995) data. because it is a relatively recent phenomenon.

The paradox pointed out by Brecher and Choudhri (1982) was confirmed for 18 of 33 countries in 1983. This suggests that the paradox matters in reality. Adjusting the trade imbalance is also important in explaining the paradox. However, whether or not the paradox can be resolved empirically depends upon the estimates of productivity.

It can be said that, when Trefler (1993) succeeded in introducing country- and factor-specific productivity into the HOV model, the paradox pointed out by Brecher and Choudhri (1982) had already been resolved. The contribution of this paper is to generalize the paradox from the case of the US to that of all countries, and to clarify this point formally and explicitly. The bottom line of this paper is that the paradox outlined by Brecher and Choudhri (1982) is now formally resolved. Note that, as discussed by Feenstra (2015), one limitation of Trefler's (1993) methodology is that there are many "free" parameters in the sense that the use of π_k^i requires $C \times M$ parameters. Note also that what this paper shows is that there is at least one possible solution to the paradox. Thus, following Maskus and Nishioka (2009), it is also important to explore alternative country-factor-specific productivity estimates. Some of these issues will be explored in the next stage of my research.

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